A report on error function

DVF and the authors of wikipedia

The report introduces the error function (also called the Gauss error function) which is a special function (non-elementary) of sigmoid shape which occurs in probability, statistics and partial differential equations.

I.   INTRODUCTION

The error function is defined as [1]

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,dt\,. \tag{1}$$

The error function is illustrated on Fig. 1. The complementary error function, denoted erfc, is defined as

$$\mathrm{erfc}(x) = 1 - \mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2}\,dt\,. \tag{2}$$

II.   THE NAME "ERROR FUNCTION"

The error function is used in measurement theory (using probability and statistics), and although its use in other branches of mathematics has nothing to do with the characterization of measurement errors, the name has stuck. The error function is used in measurement theory (using probability and statistics), and although its use in other branches of mathematics has nothing to do with the characterization of measurement errors, the name has stuck. The error function is related to the cumulative distribution Φ, the integral of the standard normal distribution, by [1].

$$\Phi(x) = \frac{1}{2} + \frac{1}{2}\mathrm{erf}(\frac{x}{\sqrt{2}})\,. \tag{3}$$

The error function, evaluated at $\frac{x}{\sigma\sqrt{2}}$ for positive x values, gives the probability that a measurement, under the influence of normally distributed errors with standard deviation , has a distance less than x from the mean value.[5] This function is used in statistics to predict behavior of any sample with respect to the population mean. This usage is similar to the Q-function, which in fact can be written in terms of the error function.
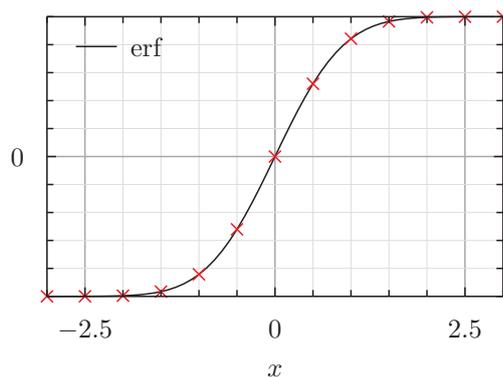

FIG. 1: The plot of the error function. The points indicate the values obtained by the `erf` function from `stdlib`.

III.   IMPLEMENTATION

The error function is implemented via the complementory error function, equation (2). For positive x the integral in equation (2) is evaluated directly by a call to the `gsl_integration_qagiv` function. For negative x the formula

$$\mathrm{erfc}(-x) = 2 - \mathrm{erfc}(x) \tag{4}$$

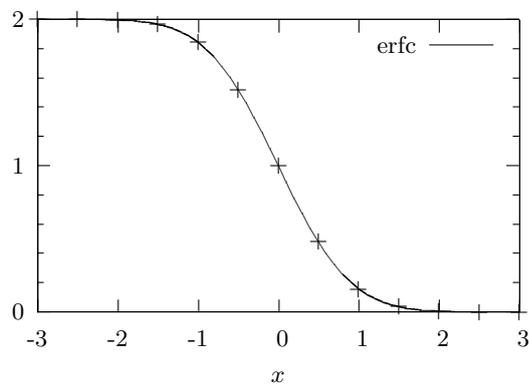was used instead.

[1] Andrews, Larry C.; Special functions of mathematics for engineers.

FIG. 2: The plot of the complimentary error function